NTNU | Institutt for matematiske fag

DERIVED REPRESENTATION TYPE AND G-EQUIVARIANT SPECTRA

Work in progress with Clover May

Jacob Fjeld Grevstad

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Overview

- History of representation type
- G-equivariant spectra and Cohomological Mackey functors
- Derived representation type and results

Representation theory of groups

- Characters of groups
  (Gauss, Dedekind, Frobenius)

1896 1911 1925 1935 1940 1954 1957 1963 1972 1977 2003

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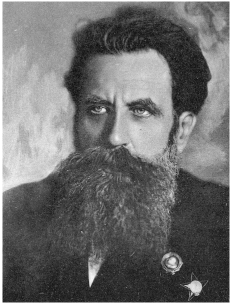
- Krull–Remak–Schmidt



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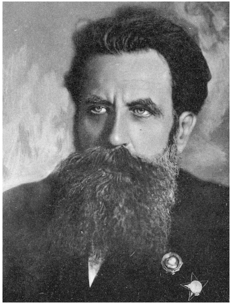
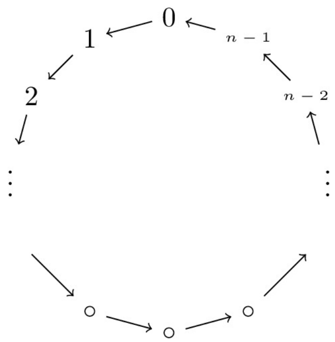
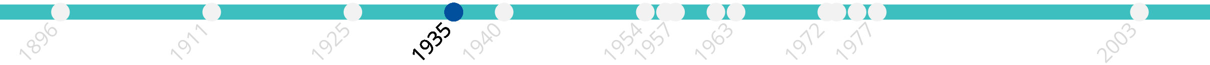


Finite type

- Uniserial rings (Köthe)





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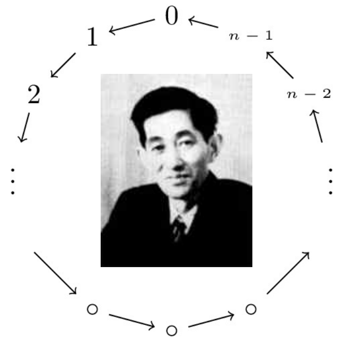
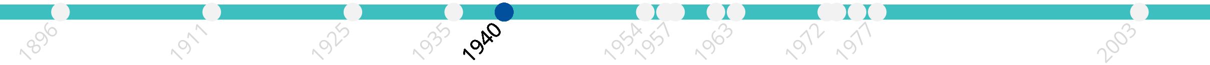
- Uniserial rings (Köthe)
- Nakayama algebras

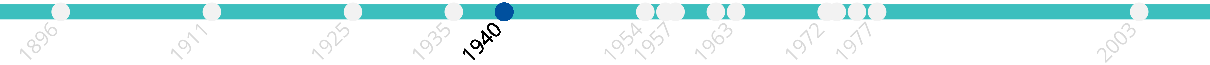




Finite type

- Uniserial rings (Köthe)
- Nakayama algebras

> Now arises the problem to determine general type of rings which possess arbitrary large directly indecomposable left or right moduli. But, the writer has to leave also this problem open; the notion of generalized uni-serial rings is, *a fortiori*, too special to settle this question.



Finite type

- Finite groups in characteristic p (Higmann)

1896 1911 1925 1935 1940 **1954** 1963 1972 1977 2003

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1896 1911 1925 1935 1940 1956 1963 1972 1977 2003

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1896 1911 1925 1935 1940 **1956** 1963 1972 1977 2003

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- Radical square zero algebras (Yoshii, Brauer, Thrall)
- Quivers and hereditary algebras (Gabriel)

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Brauer-Thrall conjectures

- Bounded type $\Rightarrow$ finite type

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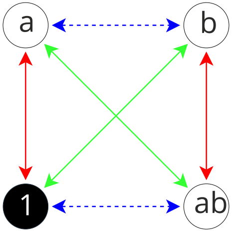
Brauer-Thrall conjectures

- Bounded type $\Rightarrow$ finite type (Roiter 1968)
- Unbounded $\Rightarrow$ Strongly unbounded (Nazarova, Roiter, Ringel 1973)

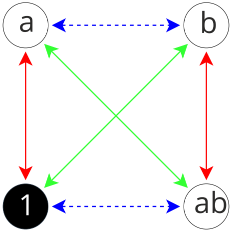
Tame type

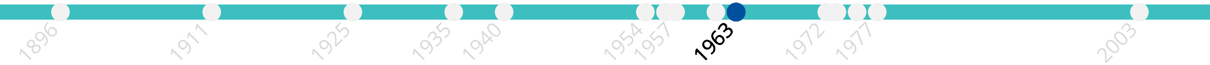
- Modules over $\overline{\mathbb{F}}_2 C_2 \times C_2$ classified (Bashev, Heller, Reiner)



Tame type

- Modules over $\overline{\mathbb{F}}_2 C_2 \times C_2$ classified (Bashev, Heller, Reiner)
- $\overline{\mathbb{F}}_p C_p \times C_p$ much harder (Krugljak)





Wild type

Definition (Donovan–Freislich)

A strict family of Λ-modules over Γ is an exact functor $F\colon \operatorname{mod}\Gamma \to \operatorname{mod}\Lambda$ such that

- F preserves indecomposability
- $FX \cong FY \implies X \cong Y$

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- F preserves indecomposability
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Definition (Donovan–Freislich)

Λ is *wild* if it has a strict family over *any* finite dimensional algebra.

Wild type

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Wild type

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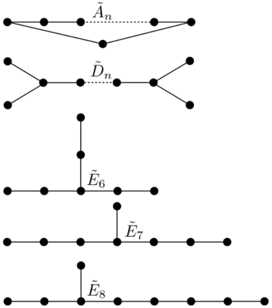

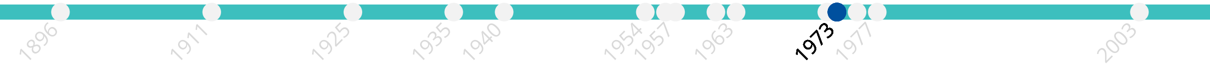

Wild type

- Wild commutative algebras (Drozd)
- Tame/Wild hereditary algebras (Nazarova)
- Wild local algebras (Ringel)

1896 1911 1925 1935 1940 1954 1957 1963 1975 2003

Wild type

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- Tame/Wild hereditary algebras (Nazarova)
- Wild local algebras (Ringel)
- Tame-Wild Dichotomy (Drozd)

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Topological interlude

Results

Theorem

If k has characteristic p, and G is a p-group different from C_2, then $\mu_k(G)$ is derived wild.

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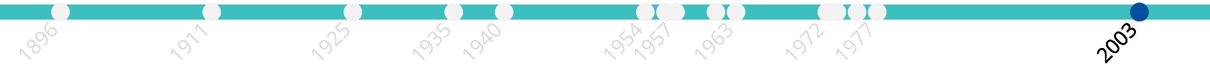
Theorem (in progress...)

If k is algebraically closed $\mu_k(G)$ is derived wild iff the sylow-p-subgroup of G is not trivial or C_2.

Derived representation type

- Gentle algebras are derived tame (Bekkert, Merklen)



Derived representation type

- Gentle algebras are derived tame (Bekkert, Merklen)
- Derived Tame-Wild Dichotomy (Bekkert, Drozd)

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- $1 \underset{\beta}{\overset{\alpha}{\rightrightarrows}} 2 \xrightarrow{\gamma} 3$ is wild

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- $1 \overset{\alpha}{\underset{\beta}{\rightrightarrows}} 2 \xrightarrow{\gamma} 3$ is wild
- $\Lambda = \mathbb{F}_2 C_2 \times C_2 = \mathbb{F}_2[s,t]/(s^2,t^2)$

$$\Lambda \otimes P_1 \xrightarrow{s\otimes\alpha + t\otimes\beta} \Lambda \otimes P_2 \xrightarrow{st\otimes\gamma} \Lambda \otimes P_3$$

$$\mathbb{F}_2^l \overset{A}{\underset{B}{\rightrightarrows}} \mathbb{F}_2^m \xrightarrow{C} \mathbb{F}_2^n \quad \mapsto \quad \Lambda^l \xrightarrow{sA+tB} \Lambda^m \xrightarrow{stC} \Lambda^n$$