Concealed g-tame algebras

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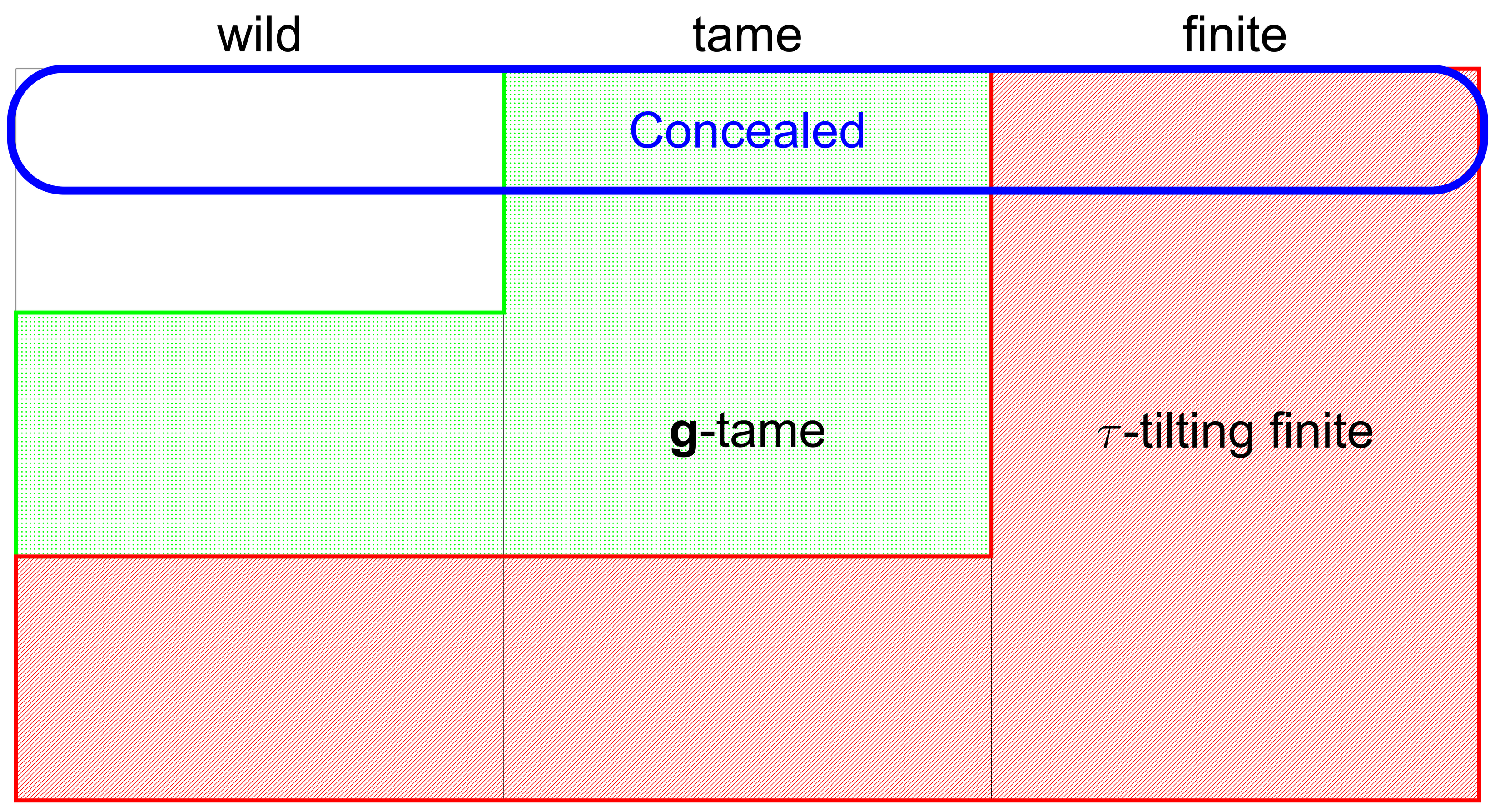
Main Theorem

Theorem *A concealed algebra is* **g***-tame if and only if it is tame.*



g-vectors

Definition A pair (M, P) in $\operatorname{mod} A \times \operatorname{proj} A$ is called τ-rigid if

- $\operatorname{Hom}(M, \tau M) = 0$
- $\operatorname{Hom}(P, M) = 0$

Definition If (M, P) is τ-rigid and $P^1_M \to P^0_M$ is a minimal projective presentation of M, then

$$\mathbf{g}_{(M,P)} := [P^0_M] - [P^1_M] - [P] \in K_0(\operatorname{proj} A)$$

is the **g**-vector of (M, P).

Definition Decompose a τ-rigid pair $(M, P) = \bigoplus_i U_i$ into indecomposables U_i. Then

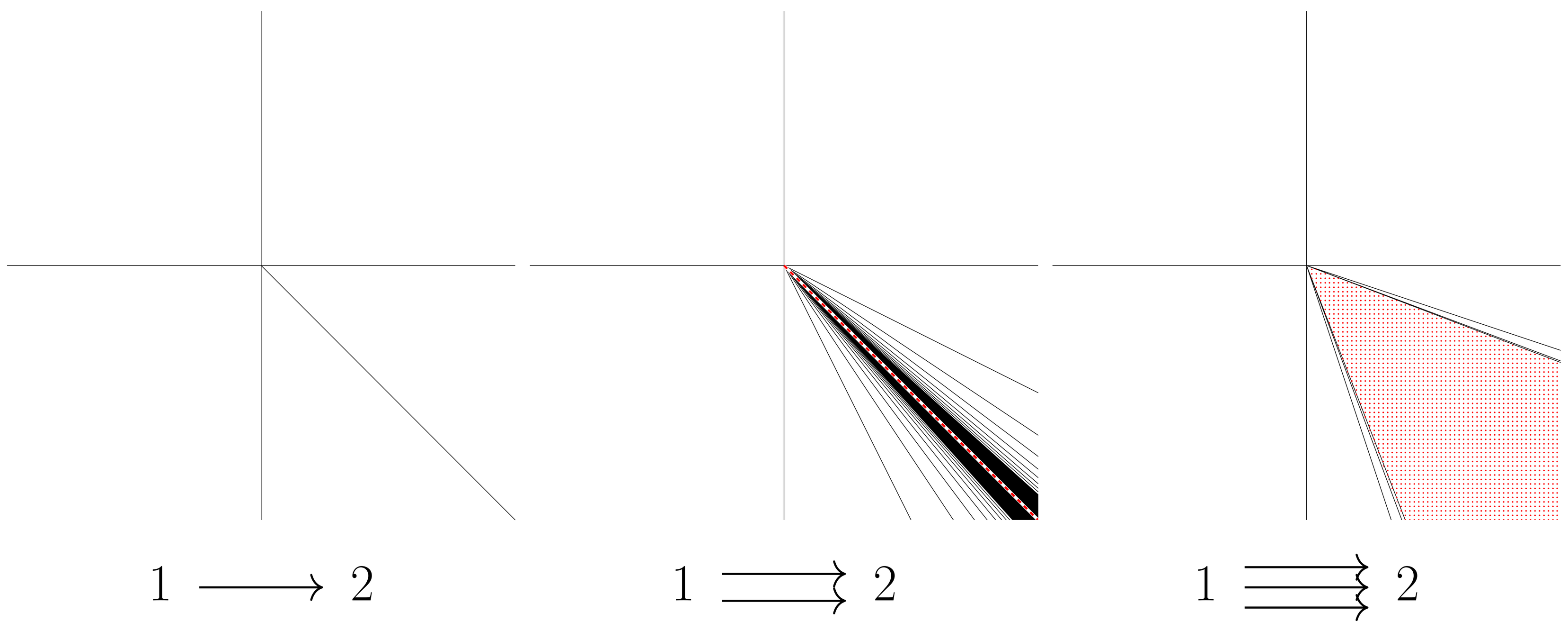
$$C^+(M, P) := \left\{ \sum_i a_i \mathbf{g}_{U_i} \mid a_i > 0 \right\} \subseteq K_0(\operatorname{proj} A) \otimes \mathbb{R}$$

is the cone associated to (M, P). The cones of all τ-rigid pairs together forms the **g**-vector fan of A.

τ-tilting type

Definition (Aoki–Yurikusa) An algebra A is called **g**-tame if its **g**-vector fan is dense in $K_0(\operatorname{proj} A) \otimes \mathbb{R} \cong \mathbb{R}^n$.

Theorem (Asai) *An algebra A is τ-tilting finite iff its* **g***-vector fan covers all of $K_0(\operatorname{proj} A) \otimes \mathbb{R} \cong \mathbb{R}^n$.*



Wall and Chambers

Definition Associated to a nonzero module M, we define a wall

$$\Theta_M := \{\theta \in K_0(\operatorname{proj} A) \otimes \mathbb{R} \;\mid\; \theta(M) = 0,\ \theta(X) \geq 0 \quad \forall X \in \operatorname{Fac} M\}$$

Theorem (Asai) *The walls are dense in the complement of the* **g***-vector fan.*

Concealed algebras

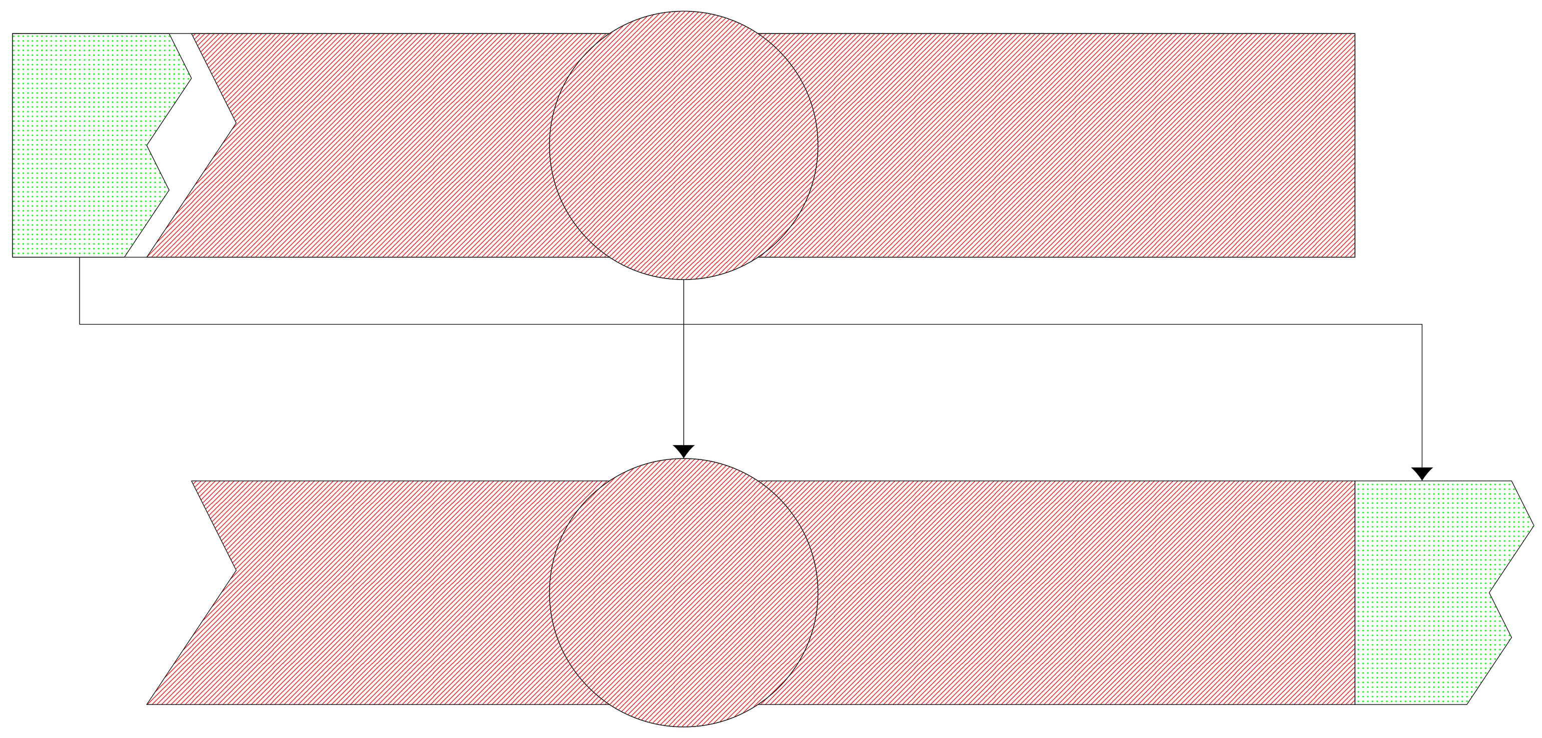
Definition (Concealed algebra) An algebra B is *concealed* of type Q if $B = \operatorname{End}_{kQ}(T)$ for a postprojective tilting kQ-module T.

Theorem (Brenner–Butler) *For a tilting kQ-module T, you have equivalences of categories*

$$\operatorname{Hom}_{kQ}(T, -) : \quad \operatorname{Fac}_{kQ} T \longrightarrow \operatorname{Sub}_B DT$$

$$\operatorname{Ext}^1_{kQ}(T, -) : \operatorname{Sub}_{kQ} \tau T \longrightarrow {}^{\perp}({}_B DT)$$

where $B = \operatorname{End}_{kQ}(T)$.



Hyperbolic algebras

Theorem *For a quiver Q, the bilinear form on $K_0(\operatorname{mod} kQ)$ given by*

$$q_Q([M]) = \dim \operatorname{Hom}_{kQ}(M, M) - \dim \operatorname{Ext}^1_{kQ}(M, M)$$

only depends on the dimension vector of M.

Theorem *Consider*

$$C_Q^{<0} := \{x \in \mathbb{R}^n \mid x > 0,\ q_Q(x) < 0\}$$

Then kQ is wild iff $C_Q^{<0}$ is nonempty.

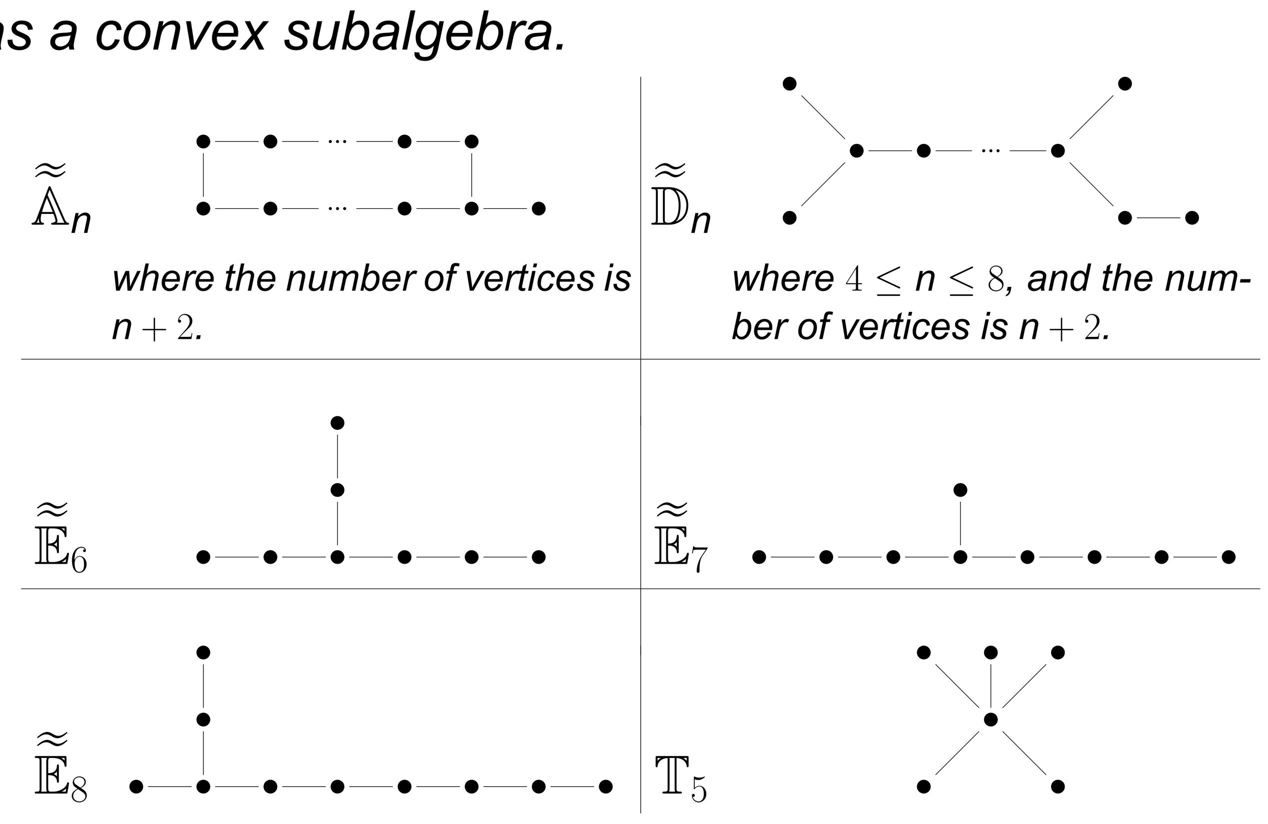
Definition A quiver Q is called *hyperbolic* if

- The path algebra kQ is wild
- For any proper full subquiver Q', the path algebra kQ' is not wild

Theorem (BGR) *Let Q be hyperbolic and $B = \operatorname{End}(T)$ be concealed of type Q. If $\theta \in C_Q^{<0}$ lies in a wall Θ_M, then the image of θ in $K_0(\operatorname{proj} B)$ lies in the wall $\Theta_{\operatorname{Hom}(T,M)}$.*

Finite posets

Theorem (Leszczyński) *A finite poset is wild if and only if its Galois cover contains a concealed algebra of one of the following types as a convex subalgebra.*



Corollary *A simply connected poset is* **g***-tame if and only if it is tame.*

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