Exact Precluster Tilting

Admissible modules

For a subcategory $\mathcal{M} \subseteq \mathcal{E}$ of an exact category we define $\operatorname{mod}_{\mathrm{adm}}(\mathcal{M}) \subseteq \operatorname{Mod}(\mathcal{M})$ as the category of functors with a presentation

$$\operatorname{Hom}_{\mathcal{E}}(-, M')|_{\mathcal{M}} \xrightarrow{f\circ -} \operatorname{Hom}_{\mathcal{E}}(-, M)|_{\mathcal{M}} \longrightarrow F \longrightarrow 0$$

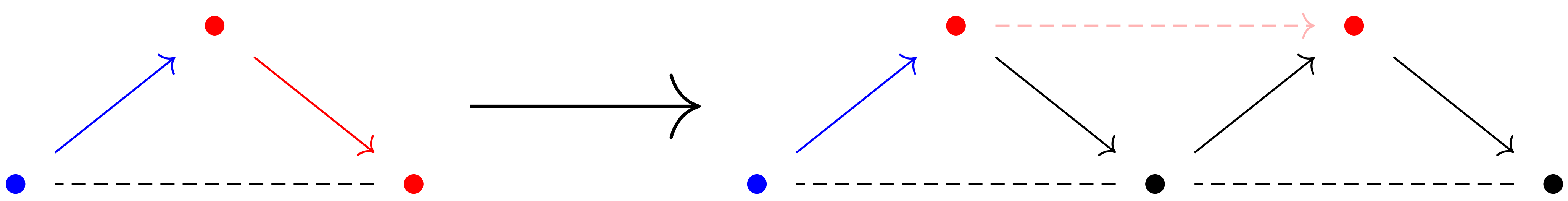
such that $f\colon M' \to M$ is admissible in $\mathcal{E}$.

> $\operatorname{mod}_{\mathrm{adm}}(\mathcal{M})$ is an exact category

> $\operatorname{mod}\Lambda \cong \Omega^2 \operatorname{mod}\Gamma$

Morita–Tachikawa correspondence

The Morita–Tachikawa correspondence gives a bijection between generator-cogenerators and algebras with dominant dimension at least 2.



When M is a generator-cogenerator over Λ and $\Gamma = \operatorname{End}(M)$, then $\operatorname{Hom}_{\Lambda}(M, -)$ is an embedding which maps injectives to projective-injectives.

Correspondences for exact categories

Mapping a subcategory to its admissible module category gives a correspondence between functorially finite generating-cogenerating subcategories and certain exact categories. This generalizes several correspondences for algebras, as illustrated in the table below.

Module	**Subcategory**	**Exact category**	**Algebra**
M	$\mathcal{M}$	$\operatorname{mod}_{\mathrm{adm}}(\mathcal{M})$	$\Gamma := \operatorname{End}(M)$
generator-cogenerator	functorially finite	enough projectives	
	generating-cogenerating	$({}^{\perp}\mathcal{P}, \operatorname{cogen}\mathcal{P})$ torsion pair	
		$X \to {}^{\perp}\mathcal{P}$ has image in ${}^{\perp}\mathcal{P}$	$\operatorname{domdim}\Gamma \geq 2$
		$\operatorname{Ext}^1({}^{\perp}\mathcal{P}, \mathcal{P}) = 0$	
n-rigid	n-rigid	$\operatorname{Ext}^{<n+1}({}^{\perp}\mathcal{P}, \mathcal{P}) = 0$	$\operatorname{domdim}\Gamma \geq n+1$
closed under τ_n and τ_n^-	relative inj.dim $< n$	$\operatorname{Ext}^{>n+1}(-, \mathcal{P}) = 0$	$\operatorname{inj.dim}_{\Gamma}\Gamma \leq n+1$
	relative cotilting	$\mathcal{P}$ cotilting	Gorenstein
$M^{\perp_{n-1}} = \operatorname{add} M = {}^{\perp_{n-1}}M$	$\mathcal{M}^{\perp_{n-1}} = \mathcal{M} = {}^{\perp_{n-1}}\mathcal{M}$	$\operatorname{Ext}^{>n+1}(-,-) = 0$	$\operatorname{gl.dim}\Gamma \leq n+1$

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